Economic application, first order differential equation

Consider a market with the following supply and demand function:

$$Q_d = 10 - 2P$$
$$Q_s = 4 + P$$

And at the same time, we know that the market price behaves in the following way over time:

$$\frac{dP}{dt} = K(Q_d - Qs)$$

Where K is a constant greater than 0.

1. Obtain the differential equation and solve to find the expression $P(t)$.
2. Find the particular solution knowing that when $t = 0$, $P = 10$.
3. What happens if $K = 0$? How does the price vary over time?

Solution

1. We combine the data to come up with the following expression:

$$\frac{dP}{dt} = K(10 - 2P - 4 + P)$$

$$\frac{dP}{dt} = 6K - PK$$

We rearrange and solve by the method of separable variables:

$$\frac{1}{K(6-P)}dP = dt$$

Integrating both sides (and assuming $P < 6$):

$$-\frac{\ln(6-P)}{K} = t + C$$

Rearranging for P:

$$6 - P = e^{-tK-CK}$$

$$P = 6 - e^{-tK-CK}$$

2. Finding the particular solution:

$$5 = 6 - e^{-CK}$$

$$e^{-CK} = 1$$

Thus $C = 0$.

$$P = 6 - e^{-tK}$$

3. If $K = 0$, then the price does not change over time, resulting in $P = 6$.